**Probing the nucleons cross section and nuclear equation of state with elliptical flow at intermediate energies**

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ABSTRACT: In this study, the detailed analysis of excitation function of elliptical flow by taking into account the nature of equation of state, nucleon-nucleon (NN) cross sections as well as momentum dependent interactions is performed within Quantum Molecular Dynamics (QMD) model. All the physical parameters are found to sensitive towards the elliptical flow. The interesting observation from the study is that the transition energy is found to be decrease with increase in the size of the fragment in the presence of static equations of state and different NN cross sections, while trend gets opposite when momentum dependent interactions are included. Finally, experimental data of INDRA collaboration around the transition energy of $^{79}$Au$^{197} + ^{79}$Au$^{197}$ is verified with static hard equation of state in the presence of large cross section.

Keywords: Elliptical Flow; Nuclear equation of state; QMD model; NN cross sections; PACS: 25.70.-z; 25.75.Ld; 24.10.Lx.

INTRODUCTION

A quite good progress has been made in the recent years in determining the nuclear equation of state from heavy-ion reactions by taking into account the phenomena of multifragmentation, collective flow as well as nuclear stopping[1–14]. Among different observables, collective flow enjoys a special status. This is due to its sensitive response to the model ingredients that define equation of state. This collective motion of the particles in heavy-ion collision can be studied via directed and elliptical flows. The directed flow measures the collective motion of the particles in the reaction plane. While, the elliptical flow is defined by the second order Fourier coefficient from the azimuthal distribution of detected particles at mid rapidity[10]. From the study using directed and elliptical flow, most of the author’s state that they did not find a unique formulation of the equation of state that reproduces all the data. Using a quantum molecular dynamics code, no definite conclusions were put forward in Refs. [4, 5, 11]. With the passage of time, a clear preference for a soft NEOS was found[12] using a Boltzmann-Uehling-Uhlenbeck (BUU) transport code[1]. A drawback of the study[12] was that conclusions were drawn from one code using one observable at one incident energy. A similar limitation holds for sub-threshold kaon production[13] which shows sensitivity[14] only in a narrow beam energy interval just below threshold and above sufficient detection probability. The most of the conclusions were based on the study of the directed flow, while the nature of equation of state using elliptical flow along a long range of beam energy is still needs a lot of research in the nuclear Community[5, 10].

On the other hand, the exact nature of nucleon-nucleon cross section is still an open question[15–18]. One has tried to fit the strength of nucleon-nucleon cross sections with the multifragmentation, while, others with the directed flow. In the literature[15], the results of multifragmentation clearly indicate that the nucleon-nucleon cross section has appreciable influence on the fragment formations at higher incident energies. The most sensitive observable to pin down the nucleon-nucleon cross section was collective flow[17, 18]. The calculations advocated its strength between 35–40 mb in QMD model[17], while, recently[19, 20] indicated its importance in term of reduced cross section in Isospin-dependent QMD model.
studies related the influence of different NN cross sections on the excitation function of elliptical flow are very a few in literature \[21\]. This motivated us to carry the analysis in detail. The momentum dependent interactions (MDI) also play a crucial role in intermediate energy heavy-ion collisions. The momentum dependence of the nuclear equation of state has been reported to affect the collective flow and particle production drastically \[3, 22–27\]. Some initial investigations also point toward its important role in multifragmentation \[23, 26\]. Recent results of Ref. \[23\] indicated enhanced production of Light Charged Particles (LCP’s) and Medium Mass Fragments (MMF’s) as compared to other type of fragments. One has to keep in mind that the response of momentum dependent interactions also depends on the system size and incident energies. For example, it has been shown by Sood and Puri \[27\] that momentum dependent interactions push energies of vanishing flow to significant lower levels for the C\(^{12}\) + C\(^{12}\) system, whereas for heavier systems, the trend is just opposite. This study motivated us to elaborate the effect of momentum dependent interactions on the excitation function of elliptical flow. Our goals are quite clear now. In this paper, we will attempt to study the sensitivity of different physical parameters in intermediate energy heavy-ion collisions using elliptical flow \(v_2\) as an observable. The physical parameters are nature of equation of state, nucleon-nucleon cross sections as well as momentum dependent interactions. We will parameterize the nuclear equation of state and nucleon-nucleon cross section by using the experimental findings of INDRA collaborations. For the present study, Quantum Molecular Dynamics (QMD) model \[28\] is used to generate the phase space of nucleons, which is discussed in Sec.2. The results and discussions are represented in Sec.3, followed by the conclusion in Sec. 4.

MATERIAL AND METHODS

The QMD model \[28–30\] is a time dependent N-body theory which simulates the time evolution of heavy-ion reactions on an event-by-event basis. It is based on the generalized variational principle. The basic assumption of QMD model is that each nucleon is represented by coherent states of the form

\[ \phi_i(r, p; t) = \left( \frac{2}{L\pi} \right)^{3/4} \exp \left[ -\frac{2}{L} (r - r_i(t))^2 + \frac{1}{\hbar} (r \cdot p_i(t)) \right], \tag{1} \]

Which are characterized by 6 time dependent parameters, \( r_i \) and \( p_i \), respectively. Here \( L \) is related to the extension of wave packet in phase space. This is found to vary between 1.08 and 2.16 fm\(^2\) \[28\]. The total N-body wave function is assumed to be a direct product of the coherent states presented in eq. 1. The equations of motion for the parameters \( r_i \) and \( p_i \) are derived using the variational principle, can be read as:

\[ \frac{d r_i}{dt} = \frac{\partial < H >}{\partial p_i} \quad ; \quad \frac{d p_i}{dt} = - \frac{\partial < H >}{\partial r_i} \quad \tag{2} \]

The Hamilton consists of the kinetic energy and effective interaction potential and is written as follow:

\[ \langle H \rangle = \sum_i \left( \frac{p_i^2}{2m_i} + \sum_{j \neq i} \int f_i V(\vec{r}, \vec{r}') f_j d\vec{r} d\vec{r}' d\vec{p} d\vec{p}' \right) \tag{3} \]

Where \( f_i \) and \( f_j \) are the function of \( (r, p, t) \) and \( (r', p', t) \). The interaction potential \( V(\vec{r}, \vec{r}') \) is based on a density-dependent Skyrme-type equation of state with additional Yukawa, Coulomb, and the Momentum dependent potentials. Mathematically, it can be written as:

\[ V(\vec{r}, \vec{r}') = \left[ t_1 \delta(\vec{r} - \vec{r}') + t_2 \delta(\vec{r} - \vec{r}')^{\gamma-1} \left( \frac{\vec{r} + \vec{r}'}{2} \right) \right] + t_3 \frac{\exp[-|\vec{r} - \vec{r}'|/\mu]}{(r' - r')/\mu} \]

\[ + \frac{Z_i Z_j e^2}{|\vec{r} - \vec{r}'|} + t_4 \ln^{2}[t_3(\vec{p}_i - \vec{p})^2 + 1] \delta(\vec{r} - \vec{r}') \tag{4} \]
The Skyrme potential consists of a sum of two- and a three-body interaction terms. The two-body term, which has linear density dependence models the long range attractive component of the NN interaction; whereas the three-body term with its quadratic density-dependence is responsible for the short range repulsive part of the interaction. The second term is parameterized in term of value instead of quadratic density dependence for N-body system. The $t_1$ and $t_2$ in Skyrme part are the constant of energy dimensions, which are parameterized in term of alpha and beta in eq.5. The $t_3$ and $\mu$ used in the Yukawa potential have the values of -6.66 MeV and 1.5 fm, respectively. In Coulomb Potential $Z_i$, $Z_j$ are the charges of i-th and j-th baryons. The momentum dependent potential constants $t_4$ and $t_5$ are parameterized in term of $\delta = 1.57$ MeV, $\epsilon = 5\times10^{-4}$ $c^2$/MeV$^2$ in eq.5. The momentum is given in units of MeV/c.

Finally, we get a parametrized density and momentum dependent single particle potential in nuclear matter, which is written as follow:

$$U = \alpha \left( \frac{\rho}{\rho_0} \right) + \beta \left( \frac{\rho}{\rho_3} \right)^\gamma + \delta \cdot \ln^2 \left( \frac{\rho}{\rho_0} \right).$$

(eq. 5)

The parameters alpha and beta are determined by demanding that at normal nuclear matter density, the binding energy should be equal to -16 MeV and must have minimal at normal nuclear matter density. The third parameter is usually treated as a free parameter for the determination of compressibility of nuclear matter. The parameterization using first two terms give rise to static soft and hard equations of state labeled as S and H. When the third term is included, it gives Soft momentum dependent (SMD) and Hard momentum dependent (HMD).

For the present calculations, the S, H, and HMD equations of state are used. The parameters for different equations of state are discussed by one of the author in Ref. [36]. For the collisions between the nucleons, the following procedure is adopted:

During the propagation, two nucleon can collide if they satisfy the condition $|r_i - r_j| < \sqrt{\sigma_{NN}/\pi}$. The phase space of scattered nucleons is checked with so called classical Pauli-blocking method. The collision process is vastly found to depend on the nucleon-nucleon cross sections. The present study is performed with energy dependent and constant cross sections [15, 16, 31, 36]. The energy dependent cross section is a parametric fit on the experimental data, which is derived by Cugnon et al [31]. Here cross section is divided into elastic and inelastic parts which depend on the center-of-mass energy available to the colliding pair of nucleons. For elastic channels, the cross section is parameterized as [31]:

$$\sigma_{NN}^{(el)}(\sqrt{s}) = \begin{cases} 55(m_b) \frac{35}{1+0.00(\sqrt{s}-1.8993)} & \text{if } \sqrt{s} < 1.8993 \\ 20 & \text{if } \sqrt{s} \geq 1.8993 \end{cases}$$

(eq. 6)

where square root of s is the nucleon-nucleon center of mass energy.

For inelastic channels, the total cross section is parameterized as

$$\sigma_{NN \rightarrow N\Delta}^{(in)}(\sqrt{s}) = \begin{cases} 0 & \text{if } \sqrt{s} < 2.015 \\ \frac{20(\sqrt{s}-2.015)^2}{0.015+0.015(\sqrt{s}-2.015)} & \text{if } \sqrt{s} \geq 2.015 \end{cases}$$

(eq. 7)

During early attempts, one has used a constant nucleon-nucleon cross section to study heavy-ion collisions through transport models. Several calculations based on a constant cross section were applied to study the disappearance of flow Following these Ref. [16], we also use a constant energy independent cross section. The nature and strength of cross-section is depicted as superscript.
RESULTS AND DISCUSSION

The elliptical flow is the center of the present study. The elliptical flow is defined as the average of the ratio of difference to sum between the square of the x and y components of the particle’s transverse momentum. Mathematically, it can be written as:

\[ v_2 - \cos(2\phi) = \frac{P_x^2 - P_y^2}{P_x^2 + P_y^2} \] (8)

Where \( p_x \) and \( p_y \) are the x and y components of the momentum. The \( p_x \) is in the reaction plane, while, \( p_y \) is perpendicular to the reaction plane. A positive value of the elliptical flow indicates in-plane enhancement of the particle emission i.e. a rotational behavior. On the other hand, a negative value of \( v_2 \) shows the squeeze out effects perpendicular to the reaction plane.

For the present study, we have simulated the reaction of \( ^{79}\text{Au}^{197} + ^{79}\text{Au}^{197} \) for thousand of events between incident energies 90 and 500 MeV/nucleon at semi-central geometry i.e. \( b = 5 \) fm. The elliptical flow is calculated in mid rapidity region ranging from -0.3 to 0.3 \([32]\) for free nucleons and light charged particles (LCPs). For the present analysis, a static hard (H) and soft (S) equations of state with and without momentum dependent interactions by varying the strength of cross section has been employed. The phase space generated by the QMD model has been analyzed using the minimum spanning tree (MST) \([28, 23]\) method. The MST method binds two nucleons in a fragment if the distance between them is less than 4 fm. The entire calculations are performed at \( t = 200 \) fm/c.

1. Effect of equation of state on excitation function of elliptical flow: In Fig. 1, we have displayed the excitation function of elliptical flow for free and light charged particles (LCPs) for \( ^{79}\text{Au}^{197} + ^{79}\text{Au}^{197} \) reaction at \( b = 5 \) fm. The importance of equations of state is studied by taking into account the hard as well as soft equation of state at constant cross section \( = 40 \text{mb} \). First of all, we will discuss the excitation function of elliptical flow and then effect of equations of state on it.

![Graph showing excitation function of elliptical flow](image)

**Fig. 1.** Incident energy dependence of elliptical flow for \( ^{79}\text{Au}^{197} + ^{79}\text{Au}^{197} \) system. The different panels are for different fragments. Here different equations of state are taken into account.

With the increase in the incident energy, elliptical flow \( v_2 \) changes from positive to negative values exhibiting a transition from the in-plane to out-of-plane emission of nucleons. The incident energy at which elliptical flow approaches to zero is dubbed as transition energy. This is because of the competition between mean field and NN collisions. This competition between the mean field and NN collisions depends strongly on the effective interactions that lead to the different transition energy due to different equations of state.
After a particular incident energy, the out-of-plane emission for $2 \leq A \leq 4$ fragments starts decreasing, leading to the valley shape for the excitation function of elliptical flow. This is due to faster movement of the spectator matter after $v_2$ reaches the maximal negative value [8]. This trend is in agreement with experimental findings of different collaborations [8]. Similar results and trends have also been reported by Zhang et al. as well as our group in their recent communication [10, 21].

If one sees the effect of fragment size on the elliptical flow for a particular equation of state (soft or hard), it is observed that elliptical flow is becoming more negative for LCP’s as compared to free particles. This is indicating more squeeze-out for LCP’s. This is due to the reason that free particles (as they are unbounded) have a possibility to enjoy in-plane as well as out-of-plane, but LCP’s (as they are bounded) have a major contribution from the out-of-plane. This can be further clarifying from the Ref. [34], where phase space of free nucleons and LCP’s is displayed. It is already shown by us in [10] that transition is possible only for free particles/LCP’s which comes from the participant zone and not for the intermediate mass fragments (IMF’s). The effect of size of fragment on elliptical flow was also shown by Zhang et al. at 100 MeV/nucleon [33]. As the size of fragment is found to affect the elliptical flow, so it is also supposed to affect the transition energy, which is discussed later on.

Let us understand the effect of equations of state on the excitation function of elliptical flow. More squeeze out is observed in the presence of hard equation of state as compared to the soft one. This is due to the higher compressibility for hard equation of state (380 MeV) as compared to soft one (200 MeV). It is in supportive with the observations from literature [23] that numbers of collisions get enhanced in the presence of hard equation of state. The effect of equations of state is more with increase in the size of the fragment. Ex: At $E = 300$ MeV/nucleon, the $((v_2)_{\text{Soft}}^{(4)} - (v_2)_{\text{Hard}}^{(4)})$ values for free, $1 \leq A \leq 4$ and $2 \leq A \leq 4$ are 0.027, 0.030 and 0.060, respectively. Moreover, the experimental transition energy for $^{79}$Au$^{197}$ + $^{79}$Au$^{197}$ reactions for the fragments $Z \leq 2$ is 100 MeV/nucleon, which is in well agreement with hard equation of state. It is expected that at the higher incident energies the effect of equations of state on the elliptical flow must reduce, which is shown many times in the literature for other observables such as directed flow, multifragmentation etc [4, 16 and 23].

2. Effect of NN cross sections on the excitation function of elliptical flow: To study the effect of NN cross sections on the elliptical flow, in Fig. 2, the incident energy dependent of elliptical flow is displayed for the fragments using energy dependent Cugnon as well as constant cross section (40 mb) in the presence of static hard equation of state. The general trend for the excitation function of elliptical flow is similar as discussed in the above section by us and others [10, 21 and 33]. It is observed from the figure that more squeeze is obtained in the presence of 40 mb cross section as compared to the Cugnon cross section.

![Fig. 2](image-url)

**Fig. 2** Same as in Fig. 1, but for different nucleon-nucleon cross sections.

If one see the excitation function of directed flow in the presence of different NN cross sections in the literature [4], it is observed that directed flow is changing from negative to positive value at earlier
incident energies with increase in the NN cross sections. It is concluded from there that with increase in NN cross sections, the dominance of NN collisions increases and hence earlier transition. Following the similar Physics for the excitation energy dependence of elliptical flow, the rotational behavior due to mean field changes to the collective expansion due to NN collisions at earlier incident energies in the presence of larger cross sections. As discussed in the above section that effect of equations of state is dominating with increase in size of the fragment is also true with increase in the magnitude of the NN cross sections. It is clear from the magnitude of ($(v_2)_{\text{HardCug}} - (v_2)_{\text{Hard40}}$) which is 0.017, 0.020 and 0.033 for the fragments. Interestingly, hard equation of state with cross section 40 mb is approaching more towards the experimental transition energy of the different collaborations [8].

3. Effect of momentum dependent interactions on the excitation function of elliptical flow: Our last aim was to study the effect of momentum dependent interactions on the excitation function of elliptical flow. For this, in Fig. 3, we have displayed the incident energy dependence of elliptical flow with and without momentum dependent interactions in the presence of hard equations of state with 40 mb cross section.

![Fig. 3](image.png)

The squeeze out is found to decrease in the presence of momentum dependent interactions. This is due to the reason that in the presence of momentum dependent interactions, the mean free path increases [28]. One can see more evidence for fewer squeezes in the Ref. [27], where number of collisions is found to decrease in the presence of momentum dependent interactions. On the other hand, elliptical flow of fragments is also found to affect drastically in the presence of momentum dependent interactions. For the free particles, almost negligible effect of momentum dependent interactions is observed, but as one move towards the LCP’s, dominant effect of momentum dependent interactions is observed. For example At $E = 300$ MeV/nucleon, the ($((v_2)_{\text{Hard40}} - (v_2)_{\text{HMD40}}$) values are 0, 0.006, 0.021 with increase in the size of the fragment. This is reported in the literature by us and others [23] that multiplicity of LCP’s and medium mass fragments (MMF’s) get enhanced in the presence of momentum dependent interactions. This is due to the break-up of the initial correlations in intermediate mass fragments leading to the production of LCP’s and MMF’s, which is otherwise not possible in the presence of static equation of state. Hence, enhanced production of LCP’s as compared to intermediate mass fragments and free particles, will further lead to the dominant effect of momentum dependent interactions for the elliptical flow of LCP’s. Interestingly, the effect of momentum dependent interactions is found to decrease with increase in the incident energy. This is true because with increase in incident energy, the transparency in the system increases which results in appreciable reduction in the momentum effects. Hence momentum dependent and static equations of state starts will approach the same results at high incident energies [35].

4. Fragment size dependence of transition energy:
In Fig. 4, we have shown the fragment dependence of transition energy under the influence of equations of state (Fig.1), NN cross sections (Fig.2) and momentum dependent interactions (Fig.3). Recently, the system mass dependence of transition energy is displayed in Ref. [10, 21]. The transition energy is found to decrease with increase in the size of the fragment for Hard\textsubscript{Cug}, Hard\textsubscript{40} and Soft\textsubscript{40}, while opposite trend is observed with HMD\textsuperscript{40}. The trend of the earlier three input parameters are in agreement with the Ref. [10], while, no such study of transition energy with HMD\textsuperscript{40} is observed in literature. As we have observed from the Fig. 3, the effect of MDI is dominating for LCP’s as compared to free particles. This effect of MDI will force LCP’s towards the reaction plane and hence higher energy is needed for the LCP’s to move out-of-plane. This will lead to increase in the transition energy with fragments in the presence of momentum dependent interactions. If we consider the first three cases, the maximum transition energy is found for Hard\textsubscript{Cug} followed by Soft\textsubscript{40} towards Hard\textsubscript{40}. It is indicating that rotational behavior remains active up to higher energies for Hard\textsubscript{Cug}, while up to lower energies for Hard\textsubscript{40}. The Soft\textsubscript{40} has sensitivity in between the above two cases. This study is indicating the importance of equations of state, NN cross sections as well as momentum dependent interactions in the intermediate energy heavy-ion collisions.

5. Comparison with experimental findings: To provide the best NN cross section and equation of state in intermediate energy heavy-ion collisions, one has to compare the findings with experimental one. In Fig. 5, the incident energy dependence of elliptical flow for different physical parameters (cross sections and equations of state), is compared with the experimental data of INDRA collaborations. The experimental transition energy for the reaction 79\textsubscript{Au}\textsuperscript{197} + 79\textsubscript{Au}\textsuperscript{197} was calculated for the fragments Z \leq 2. Our results are with QMD model for 1 \leq A \leq 4 which contains the neutron as well as proton contribution. One can extract the protons by using the Isospin QMD model, which is recently used by us to extract the symmetry energy in term of neutron-to-proton ratio [37, 38]. From the figure, it is clear that the transition energy value of the experimental data of INDRA collaborations is well reproduced with Hard\textsuperscript{40}, while, and is found to deviate for the other input parameters followed by HMD\textsuperscript{40}, Soft\textsuperscript{40} and Hard\textsubscript{Cug}. The same deviation from the experimental data is obtained by using the soft momentum dependent interactions with Cugnon and constant cross section (not shown here). One can get the exact value as that of experimental by enhancing the value of NN cross section by using hard equation of state, while, soft equation of state is providing the indication for the reduction of the NN cross section to approach the experimental data. However, the uncertainties of 4-5% are also observed in the measurement of the elliptical flow, but, which are not going to change the conclusion of the article. The similar conclusions were obtained by Puri et al. [4, 16], when they compared the directed flow in term of balance energy with the experimental data.
The conclusion was demanding large cross section of 55 mb in the presence of hard equation of state. From the above study, it is concluded that hard equation of state with enhanced cross section without momentum dependent interactions are of great importance in intermediate energy heavy-ion collisions.

![Graph](image)

**Fig. 5** Comparison of transition energy obtained with QMD model with the experimental findings of INDRA/FOPI collaborations for $^{197}$Au+$^{197}$Au for $1 \leq A \leq 4$ fragments. The solid star represents the experimental data, while, other lines and symbols are with different equations of state and different cross sections.

**CONCLUSIONS**

In conclusion, we have presented the detailed analysis of excitation function of elliptical flow with the nature of equations of state, nucleon-nucleon cross sections as well as momentum dependent interactions by using QMD model. Interestingly, the transition energy is found to be decrease with increase in the size of the fragment in the presence of static equation of state and different NN cross sections, while, trend gets opposite when momentum dependent interactions are included. In addition, the transition energies, with increase in the size of fragment, become more sensitive towards the larger cross sections. Finally, the experimental data for the transition energy of INDRA collaborations is verified with static hard equation of state in the presence of large cross sections.

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